

Estimating the Zero-Lift Drag Coefficient from Aircraft Climb Data

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Introduction & Problem Statement

The zero-lift drag coefficient ($c_{D,0}$) is known to be a critical parameter in aircraft performance. Its estimation has been addressed in studies such as [1]. This work proposes a method for estimating $c_{D,0}$ from aircraft data during the climb flight phase, based on the *parabolic drag polar* equation $c_D = c_{D,0} + c_L^2/\pi A R e$. In this work, we assume that the $c_{D,0}$ value accounts for the *clean configuration* value plus any increment values that occur during climb.

Let F be the number of flights, with each flight f having a number of data points N_f , which differs between flights. The data points contain observed values for variables such as the aircraft's rate-of-climb (η), true airspeed (v_{tas}), air density (ρ), and aircraft's mass (m_0). Also, let the climb phase equation of motion (ODE) [2] be conveniently written in linear regression form:

Problem

$$\mathbf{r} = \theta \mathbf{x} + \varepsilon$$

Response (r)

$$\mathbf{r} = \eta^2 \dot{\eta} - \mathbf{k}_1 - \mathbf{k}_2 \eta - \mathbf{k}_3 \eta^2$$

Regressor (x)

$$\mathbf{x} = -f(\gamma, c_L) \eta^2$$

with $\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3$ the ODE constants of each flight F , respectively. Such constants depend on aerodynamic, engine and design parameters of the aircraft. The objective of this work is to obtain a flight-pooled estimate of the parameter $\theta = c_{D,0}$, with $\{\theta > 0 \mid \theta \in \mathbb{R}\}$ by use of the weighted least squares (WLS) method. That is, from all the data points of the available flights, we want a single estimate of θ .

Data

For this work, we select the Airbus A321 as an example aircraft with the selected flights $F = 850$ belonging to a same tail number (ICAO24 code) A321. The dataset used corresponds to the *Climbing Aircraft Dataset* [3], of the *Scientific Datasets* made available by the OpenSky Network¹. Aircraft parameters used for simulations are: $A = 127 \text{ m}^2$, $b = 33.9 \text{ m}$, $e = 0.78$, BPR = 5.5, $N_e = 2$, $F_{N,0} = 135 \cdot 10^3 \text{ kN}$ (CFM56-5B turbofan engine) [4].

Methodology

The WLS method is defined as:

$$\hat{\theta} = \arg \min_{\theta} (\mathbf{r} - \theta \mathbf{x})^T \mathbf{W} (\mathbf{r} - \theta \mathbf{x}) = \arg \min_{\theta} \sum_{f=1}^F \sum_{i=1}^{N_f} w_i (r_i - \theta x_i)^2$$

Having the block diagonal matrix $\mathbf{W} = (\mathbf{W}_1, \dots, \mathbf{W}_F)$, with $\mathbf{W}_f = \text{diag}(w_i)_{i \in N_f}$ the per-flight weight matrix, and $w_i = 1/\sigma_{e,i}^2$ the weights that average each points information. It is a method known to have a closed-form solution given by $\hat{\theta} = \mathbf{x}^T \mathbf{W} \mathbf{r} / \mathbf{x}^T \mathbf{W} \mathbf{x}$, which, under certain conditions, provides the linear unbiased estimator with smallest variance.

In order to apply WLS, we need an estimation for both η and $\dot{\eta}$ which is obtained by use of Gaussian Processes (GPs). It is done via the maximization of the marginal log-likelihood, where the hyperparameters lengthscale (ℓ), signal variance (σ_f) and noise (σ_n) are obtained for each flight. The kernel selected was the radial basis function (RBF), and Fig. 1 shows an example GP fit.

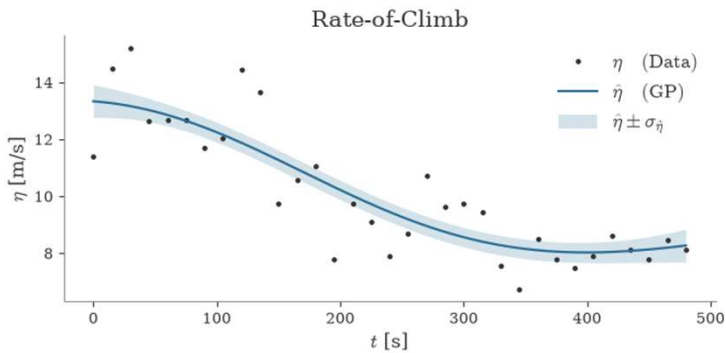


Fig. 1 Example of a GP fit on a specific segment (segment ID: 1829).

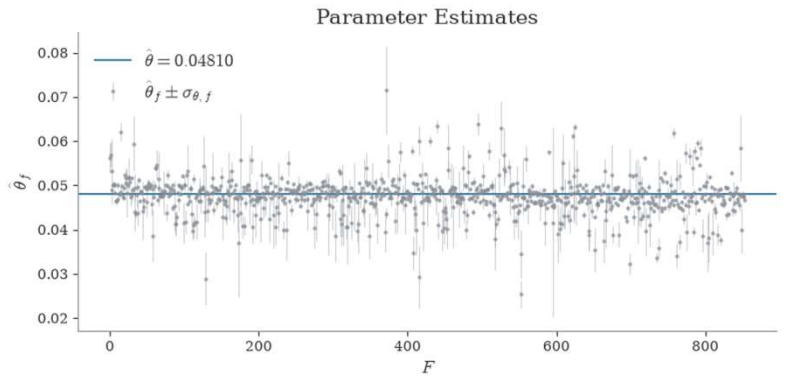


Fig. 2 Estimated per-flight parameter $\hat{\theta}_f$ with the pooled estimate.

Results & Discussion

Simulations provide an estimated $\hat{\theta}$ of 0.04810, and the per-flight estimates can be observed in Fig. 2. Our result is found to be accurate with the ones provided in [1], where the *initial climb* $c_{D,0}$ of the A321 is estimated in 0.053. Further work will focus on studying the compressibility effects that arise during climb in order to provide a climb $c_{D,0}$ estimate at a more granular level.

Bibliography

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